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Path Integral Quantization and the Ground-State Wave Functional for Multiplier Scalar-Vector Field Systems

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With the help of path-integral quantization and Fradkin's approach, we obtain a new representation in the Schrödinger picture of the multiplier scalar-vector fields and the ground-state functional. We show that the model is equivalent to free scalar fields with the same mass.

Recently, Fradkin (1993) formulated a relationship between the path-integral partition function and the absolute value squared of the ground-state wave functional in the Schrödinger representation. This method has been used to calculate the ground-state wave functional of a number of models, e.g., the Thirring–Luttinger model, coset models, and the Sutherland model (Fradkin and Moreno, 1993). In particular, it was shown that the wave functionals of the liquid ground states of fractional quantum Hall systems, in the thermodynamic limit, are universal at long distances and they have a generalized Laughlin form (Lopez and Fradkin, 1992). In previous work (Feng and Qiu, 1995) we obtained the correct ground-state wave functional for the Maxwell–Chern–Simons model using this method, and showed that Fradkin's approach can apply to singular systems without special difficulty and that the Maxwell–Chern–Simons model does not have fractional statistics. In this paper we quantize the multiplier scalar-vector fields model proposed by Li (1991) by path-integral quantization. We find that the model is equivalent to a model of free scalar fields with equal masses.

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The Lagrangian of the model considered is

$$\mathcal{L} = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + \tfrac{1}{2}m^2A^{\mu}A_{\mu} - mA_{\mu}\partial^{\mu}\varphi + \tfrac{1}{2}\partial^{\mu}\varphi\partial_{\mu}\varphi \tag{1}$$

where $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$, and φ is called the multiplier scalar field. The canonical momenta are given by definition as

$$\pi^{\mu} = \frac{\partial\mathcal{L}}{\partial\dot{A}_{\mu}} = F^{\mu 0}, \qquad \pi = \frac{\partial\mathcal{L}}{\partial\dot{\varphi}} = \dot{\varphi} - mA^0 \tag{2}$$

The Poisson brackets are

$$\{\pi^{\mu}(x), A_{\nu}(y)\}|_{x^0=y^0} = -\delta^{\mu}_{\nu}\delta(\mathbf{x} - \mathbf{y}) \tag{3}$$

$$\{\pi(x), \varphi(y)\}|_{x^0=y^0} = -\delta(\mathbf{x} - \mathbf{y}) \tag{4}$$

The canonical Hamiltonian is

$$\begin{aligned}\mathcal{H}_c &= \pi^{\mu}\dot{A}_{\mu} + \pi\dot{\varphi} - \mathcal{L}\\ &= -\tfrac{1}{2}\pi^i\pi_i + \tfrac{1}{2}\pi^2 + \tfrac{1}{4}F_{ij}F^{ij}\\ &\quad - \tfrac{1}{2}m^2A_iA^i - \tfrac{1}{2}\partial^i\varphi\partial_i\varphi + mA^i\partial_i\varphi + A_0(m\pi - \partial_i\pi^i)\end{aligned} \tag{5}$$

($i = 1, 2, \ldots, d$; the spacetime is assumed of dimension $D = d + 1$). From equation (2), we have the primary constraint

$$\mathscr{C}_1 = \pi^0 \approx 0 \tag{6}$$

so the total Hamiltonian is

$$H_T = \int d^d\mathbf{x}\, \mathcal{H}_T = \int d^d\mathbf{x}\, (\mathcal{H}_c + \lambda(x)\mathscr{C}_1) \tag{7}$$

where $\lambda(x)$ is the Lagrangian multiplier. From

$$\{\mathscr{C}_1, H_T\} \approx 0 \tag{8}$$

we have a secondary constraint

$$\mathscr{C}_2 = m\pi - \partial_i\pi^i \approx 0 \tag{9}$$

It can be checked that there exist no further constraints and the constraints $\mathscr{C}_1$, $\mathscr{C}_2$ are first class

$$\{\mathscr{C}_1, \mathscr{C}_2\} \approx 0 \tag{10}$$

So there should be two gauge-fixing conditions (Dirac, 1964). In the ($D = d + 1$)-dimensional case, the physical phase space is of dimension $2(D + 1) - 4 = 2d$. We choose the first gauge condition as the familiar Coulomb condition

$$\Omega_1 = \partial^i A_i \approx 0 \tag{11}$$

The consistent condition $\dot{\Omega}_1 \approx 0$ and the definition of π^i suggest that the second gauge condition could be chosen as

$$\Omega_2 = \partial_i \pi^i - \partial_i \partial^i A^0 \approx 0 \tag{12}$$

Since $\mathrm{Det}\{\Omega_i, \mathscr{C}_j\}$ = const, the source-free partition functional is

$$Z[0] = \int \mathscr{D}\pi^\mu \, \mathscr{D}A_\mu \, \mathscr{D}\pi \, \mathscr{D}\varphi \, \delta(\mathscr{C}_1)\delta(\mathscr{C}_2)\delta(\Omega_1)\delta(\Omega_2) \times \exp\left\{ i \int d^D x \, (\pi^\mu \dot{A}_\mu + \pi\dot{\varphi} - \mathscr{H}_c) \right\} \tag{13}$$

Because of $\delta(\mathscr{C}_2)$, $\delta(\Omega_1)$, the last term and the term $A^i \partial_i \varphi$ in $\mathscr{H}_c$ can be neglected. Since

$$\delta(\Omega_2) = \mathrm{const} \cdot \delta(A_0 - \nabla^{-2} \partial_i \pi^i) \tag{14}$$

π^0 and A_0 can be integrated first,

$$Z[0] = \int \mathscr{D}\pi^i \, \mathscr{D}A_i \, \mathscr{D}\pi \, \mathscr{D}\varphi \, \delta(\mathscr{C}_2)\delta(\Omega_1) \times \exp\left\{ i \int d^D x \left(\pi^i \dot{A}_i + \pi\dot{\varphi} + \frac{1}{2}\pi^i\pi_i - \frac{1}{2}\pi^2 - \frac{1}{4}F^{ij}F_{ij} + \frac{1}{2}A_i A^i + \frac{1}{2}\partial^i\varphi\partial_i\varphi \right) \right\} \tag{15}$$

After integrating π, we have

$$Z[0] = \int \mathscr{D}\pi^i \, \mathscr{D}A_i \, \mathscr{D}\varphi \, \delta(\Omega_1) \times \exp\left\{ i \int d^D x \left[\pi^i \left(\dot{A}_i - \frac{1}{m}\partial_i\dot{\varphi} \right) + \frac{1}{2}\pi^i K_{ij} \pi^j - \frac{1}{4}F^{ij}F_{ij} + \frac{1}{2}A_i A^i + \frac{1}{2}\partial^i\varphi\partial_i\varphi \right] \right\} \tag{16}$$

where $K_{ij} = \eta_{ij} + (1/m^2)\partial_i\partial_j$. Integrating π^i gives

$$Z[0] = \int \mathscr{D}A_i \, \mathscr{D}\varphi \, \delta(\Omega_1) \exp\left\{ i \int d^D x \left[-\frac{1}{2}\left(\dot{A}_i - \frac{1}{m}\delta_i\dot{\varphi} \right) \times \Delta^{ij}\left(\dot{A}_j - \frac{1}{m}\partial_j\dot{\varphi} \right) - \frac{1}{4}F^{ij}F_{ij} + \frac{1}{2}A_i A^i + \frac{1}{2}\partial^i\varphi \, \partial_i\varphi \right] \right\} \tag{17}$$

where

$$\Delta^{ij} = \eta^{ij} - \frac{\partial^i \partial^j}{m^2 - \nabla^2}, \qquad \nabla^2 = -\partial^i \partial_i$$

Because of $\delta(\Omega_1)$, the first term in the exponential is

$$-\frac{1}{2}\left(\dot{A}_i - \frac{1}{m}\,\partial_i\dot{\varphi}\right)\Delta^{ij}\left(\dot{A}_j - \frac{1}{m}\,\partial_j\dot{\varphi}\right) = -\frac{1}{2}\left(\dot{A}^i\dot{A}_i + \frac{1}{m^2}\,\partial_i\dot{\varphi}\Delta^{ij}\partial_j\dot{\varphi}\right) \quad (18)$$

Since

$$-\frac{1}{2m^2}\,\partial_i\dot{\varphi}\Delta^{ij}\partial_j\dot{\varphi} = \frac{1}{2}\,\dot{\varphi}\,\frac{-\nabla^2}{m^2 - \nabla^2}\,\dot{\varphi} \quad (19)$$

we may define a new scalar field ϕ which is a new representation of the original field φ,

$$\phi = \sqrt{\frac{-\nabla^2}{m^2 - \nabla^2}}\,\varphi \quad (20)$$

So

$$-\frac{1}{2m^2}\,\partial_i\dot{\varphi}\Delta^{ij}\partial_j\dot{\varphi} + \frac{1}{2}\,\partial^i\varphi\partial_i\varphi = \frac{1}{2}\,[\dot{\phi}^2 + \phi(\nabla^2 - m^2)\phi] \quad (21)$$

Hence

$$Z[0] = \int \mathfrak{D}A_{iT}\,\mathfrak{D}\phi\,\exp\left\{i\int d^Dx\left[-\frac{1}{2}\,\dot{A}_{iT}\dot{A}^i_T - \frac{1}{4}\,F^{ij}F_{ij} + \frac{1}{2}\,m^2A_{iT}A^{iT} + \frac{1}{2}\,\dot{\phi}^2 + \frac{1}{2}\,\phi(\nabla^2 - m^2)\phi\right]\right\} \quad (22)$$

where $\mathbf{A}_T$ is the transverse part of $\mathbf{A}$: $A_i = (\eta_{ij} + \partial_i\partial_j/\nabla^2)A^j$. We can rewrite $Z[0]$ as

$$Z[0] = \int \mathfrak{D}A_{iT}\,\mathfrak{D}\phi\,\exp\left\{-\frac{1}{2}\,i\int d^Dx\,[\mathbf{A}_T(\partial^\mu\partial_\mu + m^2)\mathbf{A}_T + \phi(\partial^\mu\partial_\mu + m^2)\phi]\right\} \quad (23)$$

Incorporating external sources $\mathbf{J}_T$, J, $\partial_i J_T^i = 0$, we have

$$Z[\mathbf{J}_T, J] = \int \mathcal{D}A_{iT}\, \mathcal{D}\phi \exp\left\{ i \int d^D x \left[-\frac{1}{2} \mathbf{A}_T(\partial^\mu \partial_\mu + m^2)\mathbf{A}_T - \frac{1}{2} \phi(\partial^\mu \partial_\mu + m^2)\phi + J_T^i A_{iT} + J\phi \right] \right\} \tag{24}$$

The absolute value squared of the ground-state wave functional is (Fradkin, 1993; Feng and Qiu, 1995)

$$|\Psi_{gs}[\mathbf{A}_T, \phi]|^2 = \int \mathcal{D}\mathbf{J}_T(\mathbf{x})\, \mathcal{D}J(\mathbf{x}) \times \left(\exp\left\{ -i \int d^d\mathbf{x}\, [J_T^i(\mathbf{x})A_{iT}(\mathbf{x}) + J(\mathbf{x})\phi(\mathbf{x})] \right\} \right) Z_{t_0} \tag{25}$$

where

$$Z_{t_0} = Z[\mathbf{J}_T, J]|_{\mathbf{J}_T(x) = \mathbf{J}_T(\mathbf{x})\delta(x^0 - t_0), J(x) = J(\mathbf{x})\delta(x^0 - t_0)}$$
$$= \exp\left\{ -\frac{1}{2} i \int d^d\mathbf{x}\, d^d\mathbf{y}\, [J_T^i(\mathbf{x})G_{ij}(\mathbf{x} - \mathbf{y})J_T^j(\mathbf{y}) + J(\mathbf{x})G(\mathbf{x} - \mathbf{y})J(\mathbf{y})] \right\} \tag{26}$$

$$G(\mathbf{x} - \mathbf{y}) = -\frac{i}{(2\pi)^d} \int d^d\mathbf{k}\, \frac{e^{i\mathbf{k}(\mathbf{x}-\mathbf{y})}}{2\omega(\mathbf{k})}, \qquad G_{ij}(\mathbf{x} - \mathbf{y}) = -\eta_{ij} G(\mathbf{x} - \mathbf{y}) \tag{27}$$

and $\omega(\mathbf{k}) = \sqrt{\mathbf{k}^2 + m^2}$. So

$$|\Psi_{gs}[\mathbf{A}_T, \phi]|^2 = N \exp\left\{ -2 \int d^d\mathbf{x}\, (\phi\sqrt{-\nabla^2 + m^2}\phi + \mathbf{A}_T\sqrt{-\nabla^2 + m^2}\mathbf{A}_T) \right\} \tag{28}$$

It can be easily checked that

$$\Psi_{gs}[\mathbf{A}_T, \phi] = \mathcal{N} \exp\left\{ -\int d^d\mathbf{x}\, (\phi\sqrt{-\nabla^2 + m^2}\phi + \mathbf{A}_T\sqrt{-\nabla^2 + m^2}\mathbf{A}_T) \right\} \tag{29}$$

It can be seen from equation (24) that the model is equivalent to d free scalar fields with the same mass m. In the case $d = 2$, we may define a scalar field associated with $\mathbf{A}_T$,

$$A_T^i = \epsilon^{ij} \frac{\partial_j}{\sqrt{-\nabla^2}} \phi'$$

The model is then reduced to a model of free fields ϕ', ϕ with equal masses m.

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